Making Sense of Discounting

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Policy Instrument Course

March 2014

(joint work with C Azar, M Hoel and M Persson, and OJS
And Arrow et al....)

First time I remember thinking about discounting

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Nuclear waste twice as expensive!

Comparing Nuke Wind

- Costs:
- Plant x
- Fuel y
- Labour z
- Waste w

Comparing Nuke Wind

- Costs:
- Plant x
- Fuel y
- Labour z
- Waste 2w

First time I remember thinking about discounting

Nuclear waste twice as expensive!

Never mind, build it 2060 instead of 2050!

Comparing Nuke Wind

- Costs:
- Plant x
- Fuel y
- Labour z
- Waste 2w

Comparing Nuke Wind

- Costs:
- Plant x
- Fuel y
- Labour z
- Waste w

Lend me a piece of paper

- Fold it 40 times

To the moon!

$2^{10} = 1000$

$2^{40} = 10^{12}$

10^8 metres

Stern Review

- Climate Change the biggest externality in human history.
- 5-20% of future GDP
- Enormous uncertainties in calculation:
- Feedback from cloudformation
- Feedback from methan release
- Feedback from ice-melting (Albedo)
- Guess which is biggest?

Stern Review

- Climate Change the biggest externality in human history.
- 5-20% of future GDP
- Enormous uncertainties in calculation:
- Feedback from cloudformation
- Feedback from methan release
- Feedback from ice-melting (Albedo)
- **DISCOUNT RATE!**

$(1+r)^{-t}V_t$

Conventional Discounting

- If some cost or benefit component at a future date t is of the magnitude V_t and the discount rate is r, the present value is
- $$(1+r)^{-t}V_t$$

The effect is big

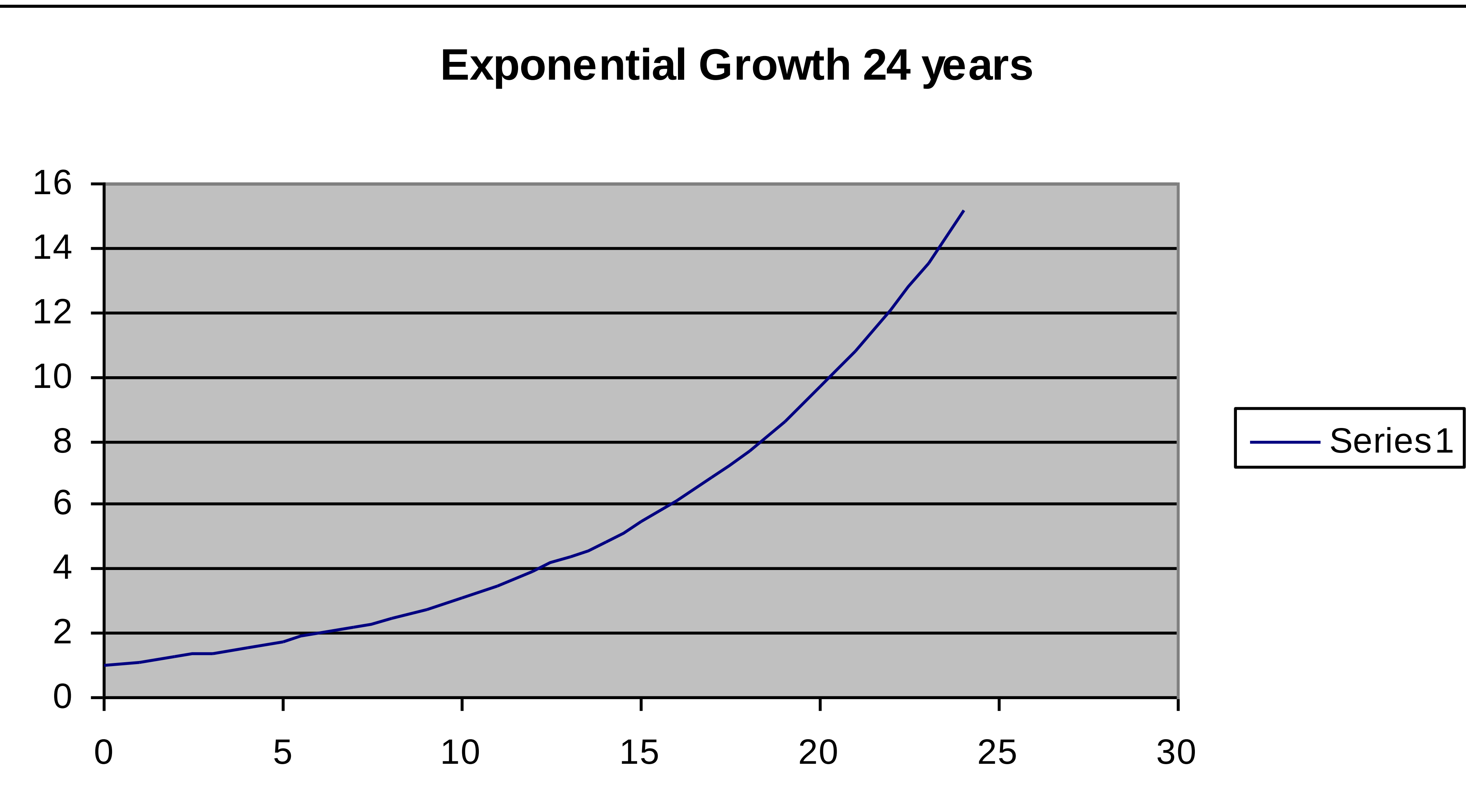
- If climate change causes a cost of 1 billion in 400 years time this is valued at 3 dollars today (5%). Had it been the same cost in 500 years then the cost would be 2 cents.
- With 6% it would have been .02 cents instead. The difference between 5 and 6 percent is thus a factor 100!

PROBLEM ?!

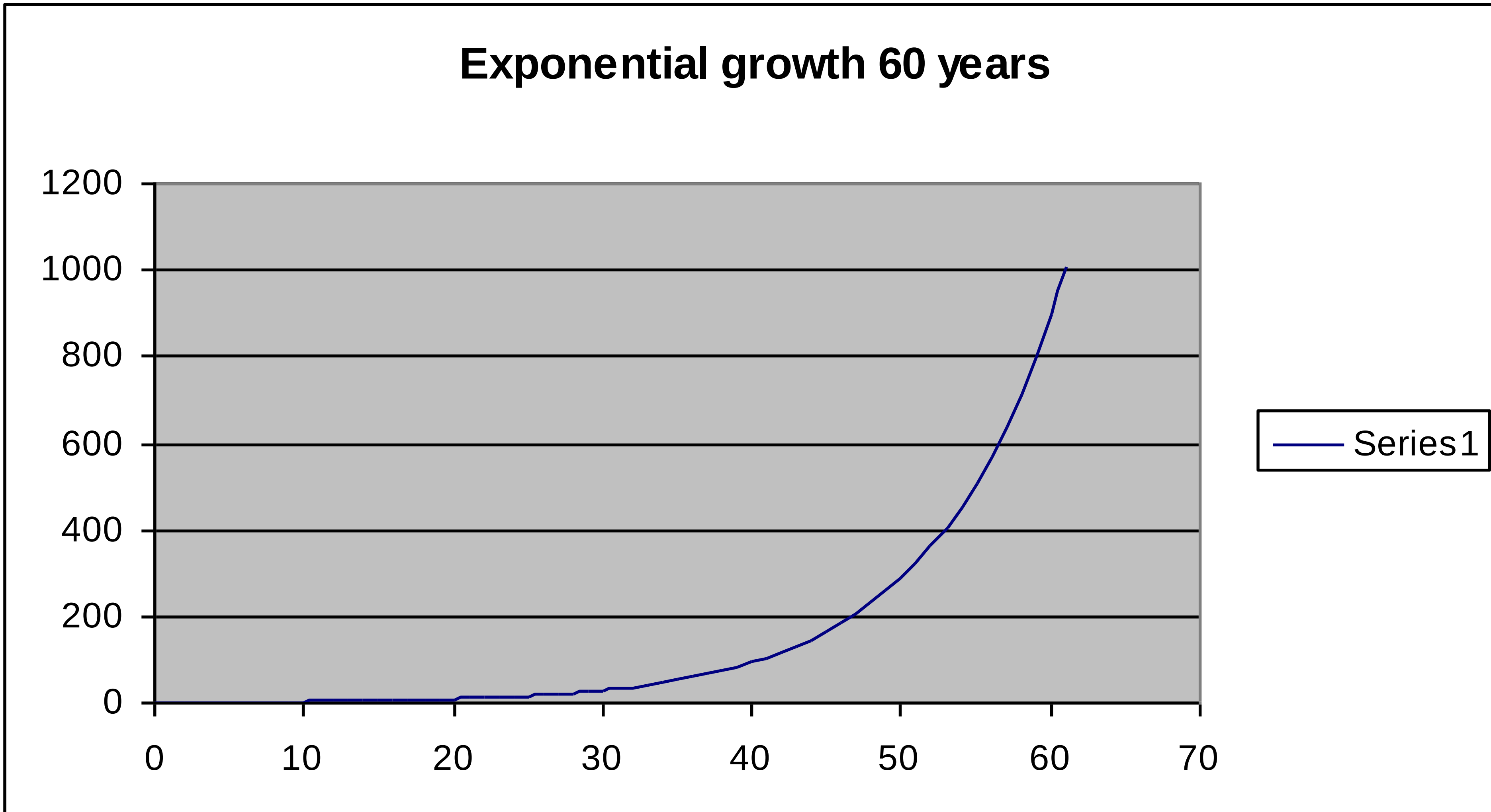
- 1$ in bank today = 2$ in 6 years
- so $2 cost in 6 years ~=~ cost of $1 today

- How big in 24 years?
- Or 240 years ie 40 doubblings – like paper

24

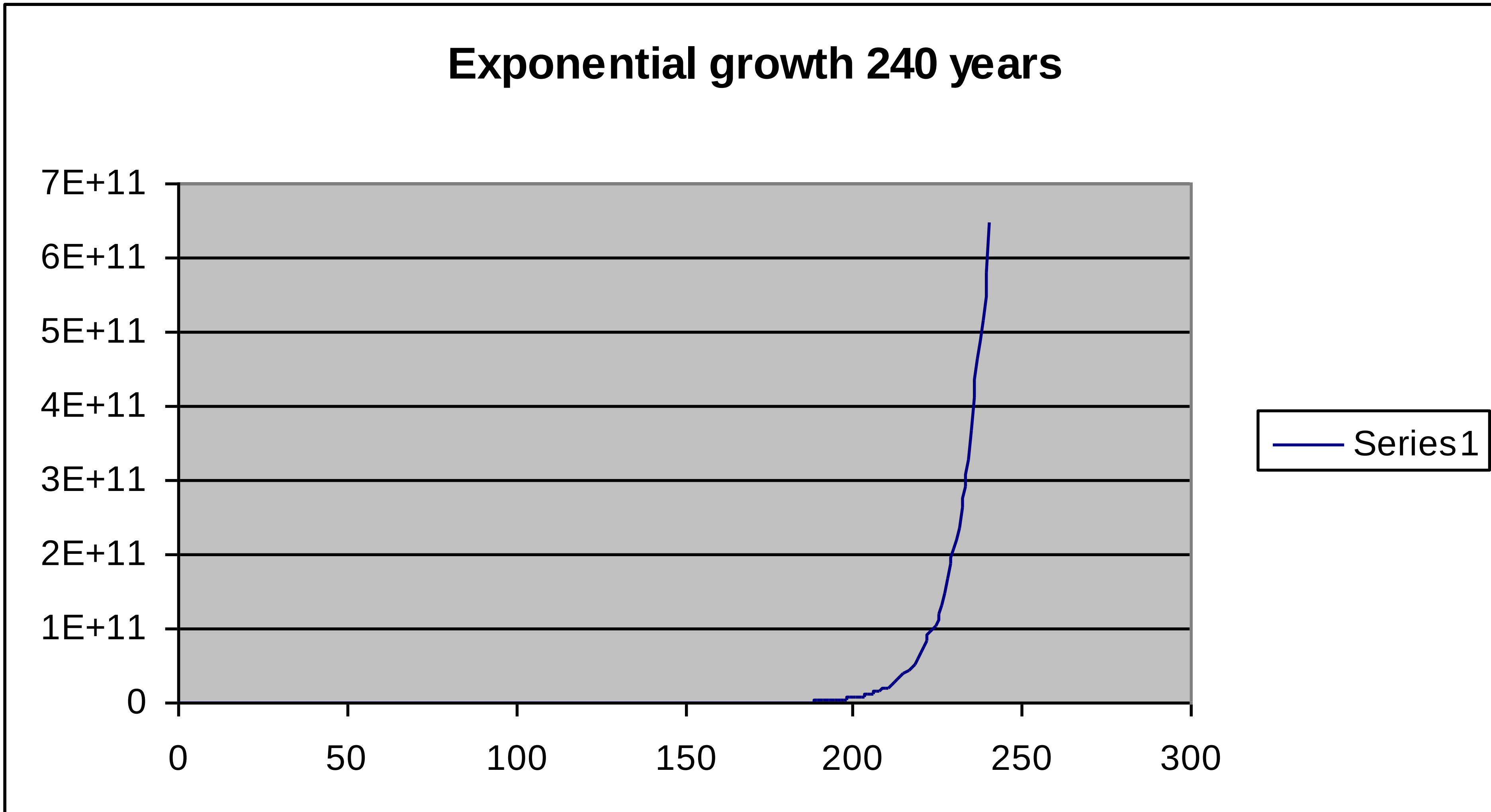

Exponential Growth 24 years
16
14
12
10
8
6
4
2
0
0
5
10
15
20
25
30
Series1

60

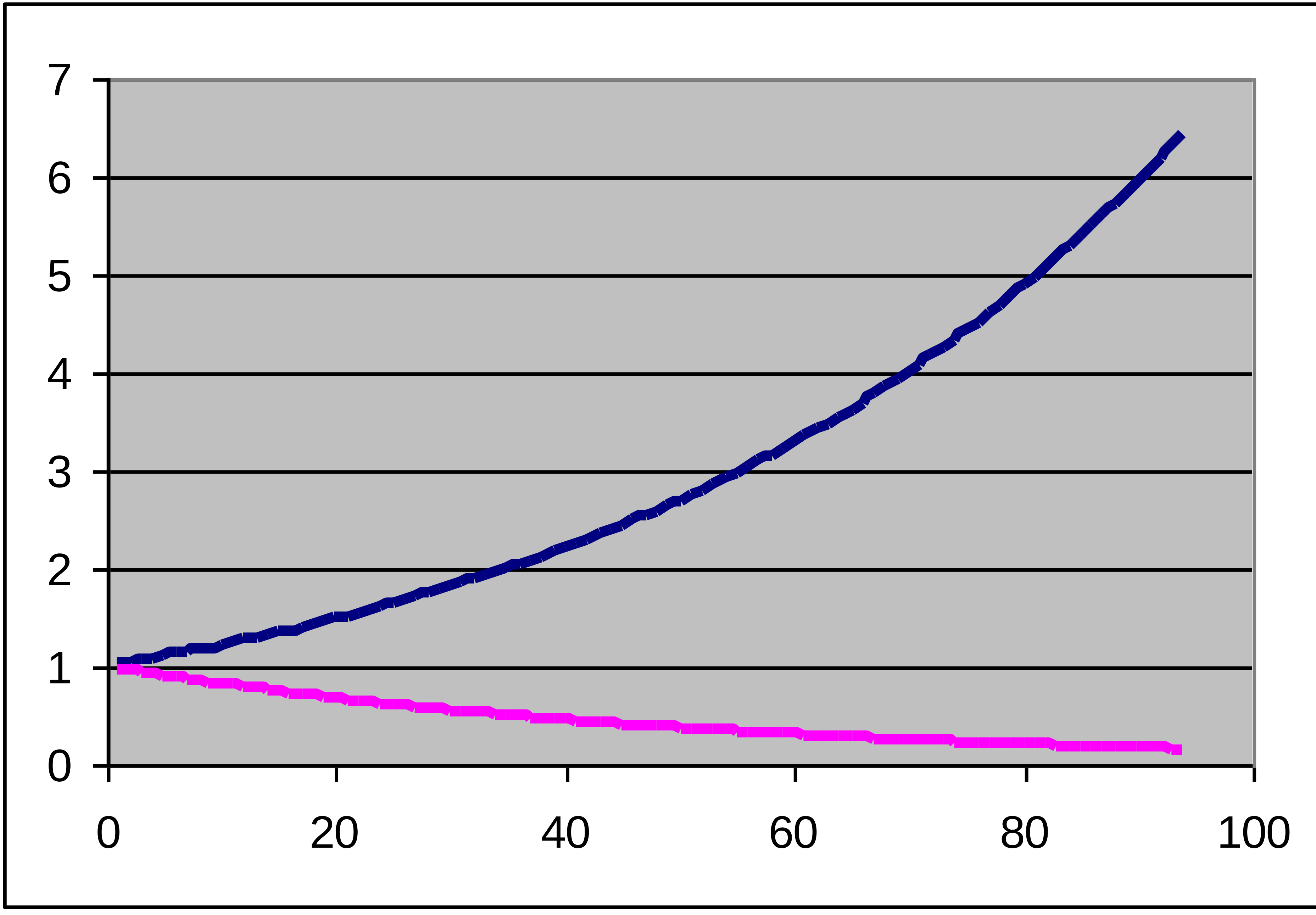


Exponential growth 60 years
1200
1000
800
600
400
200
0
0
10
20
30
40
50
60
70
Series1

240



Exponential growth 240 years
7E+11
6E+11
5E+11
4E+11
3E+11
2E+11
1E+11
0
0
50
100
150
200
250
300
Series1


7
6
5
4
3
2
1
0
0
20
40
60
80
100

Many Issues

- Can growth continue forever?
- Psychological aspects
- Hyperbolic and Gamma Discounting
- Risk

- RELATIVE PRICES

Correct value of future project

- $V_t = V_o(1+r)^{-t}(1+p)^t$
- The effect of relative prices can be as big as discounting!
- If p is big enough?

Example Land

- Property in London 19%; Scotland 11%
- Flooding of London will be costly

Labour

- 100 years ago 10% of the population in New York had a maid.
- Incomes are growing 5%/year

Labour

- 100 years ago 10% of the population in N York had a maid.
- Incomes are growing 5%/year

- How many people have a maid today?

Why can’t we all have maids?

Why can't we all have maids?

- $P_{maid} = f\ (\text{Income})$

FOOD

- World Agriculture is 24% GDP

- Lets assume we loose 1% of World Agriculture. How big is loss?
- Roughly 0.01*0.24 = = 0. 24 % GDP

FOOD

- World Agriculture is 24% GDP

- Now assume we loose 95% of World Agriculture. How big is loss?
- Roughly 0.95*0.24 = 23 % GDP

FOOD

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- Now assume we loose 95% of World Agriculture. How big is loss?
- Roughly 0.95*0.24 = 23 % GDP

- **23%! Doesnt seem right does it**
- **But what is wrong?**

Relative Prices of food…

Relative Prices of food…

- will change so fast
- That the 5% left which today accounts for 1% of GDP will become ALL of GDP.

Future Ecosystem Scarcities

- Water
- Soil
- Wild (non-cultivated) fish
- Biodiversity
- Glaciers and snow
- Wildlife, protected areas
- Fuelwood, pasture, silence (?)

OK: Economics

- Why do we discount?

OK: Economics

- Why do we discount?

- We will be richer

- We are impatient

- Rich people dont know the value of money

Assume an intertemporal welfare function

$$W = \int_0^T e^{-\rho t} U(C(t))dt$$

The tradeoffs between consumption at different points of time are given partly by the “utility discount rate” ρ

partly by the utility function *U*.

The appropriate discount rate is the sum of these two reasons

$$r = \rho - \frac{\frac{d}{dt} U'(C(t))}{U'(C(t))}$$

With Constant elasticity of utility function → classical Ramsey Rule

$$U(C) = \frac{1}{1-\alpha} C^{1-\alpha}$$

$$r(t) = \rho + \alpha g_C(t)$$

Ramsey and growth

- If $\rho = 0.01$, $\alpha = 1.5$ and $g = 2.5\%$ $r = 4.75\%$.
- Constant over time iff growth is constant.
- Increases with growth
- If growth falls, future discount rates will fall over time. Azar & Sterner (1996): limits to growth → falling discount rates and higher damage from carbon emissions.

Compare Nordhaus 5 $/ton

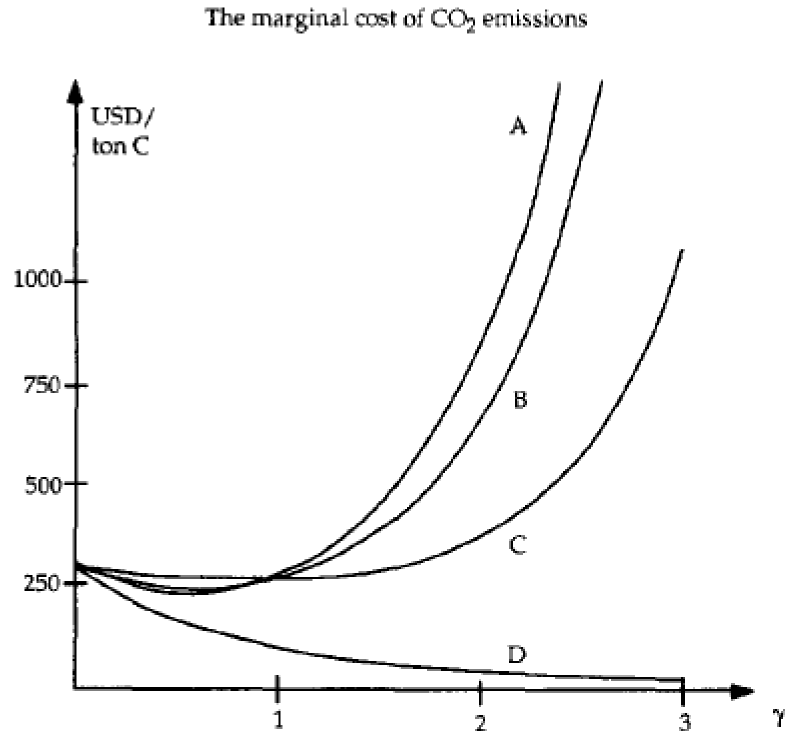


Fig. 3. The *generalized* cost of a unit emission of CO_2 is plotted as a function of γ in four cases. In plot A, B and C, the inequality situation is worsened, unchanged, and improved, respectively. In plot D, income distribution is not considered. The higher the value for γ, the higher is the discount rate, but also the inequality aversion.

Are there Limits to Growth?

- Clearly YES:
- A finite planet
- The amount of cement, carbon, steel and water that we can use is limited!

Are there Limits to Growth?

- Clearly YES:
- A finite planet
- The amount of cement, carbon, steel and water that we can use is limited!
- Clearly NO:
- Human imagination is limitless
- The quality of concerts and computer games knows no bounds!

Our best image of the future

- Continued growth…
- Rich get even richer.
- Poor will eventually also get richer but gap not eliminated.
- Much of growth in manufactured goods that use little resources. More mobiles, culture, computation, communication…
- Less transport, corals, clean water?

We need two sectors:
C which grows; E (which does not)

$$W = \int_0^\infty e^{-\rho t} U(C,E)\,dt$$

The appropriate discount rate *r* is then

$$r = \rho + \frac{-\frac{d}{dt} U_C(C,E)}{U_C(C,E)}$$

Relative price of ”environment”

Value of environmental good is given by

$$U_E / U_C$$

The relative change in this price, *p*, is

$$p = \frac{\frac{d}{dt}\left(\frac{U_E}{U_C}\right)}{\left(\frac{U_E}{U_C}\right)}$$

To simplify: select utility function that combines contant elasticity of utility above with constant elasticity of substitution between E and C

$$U(C,E)=\frac{1}{1-\alpha}\left[(1-\gamma)C^{1-\frac{1}{\sigma}}+\gamma E^{1-\frac{1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{\sigma-1}}$$

The relative price effect

$$p = \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{U_E}{U_C}\right)}{\left(\frac{U_E}{U_C}\right)} = \frac{1}{\sigma}(g_C - g_E).$$

Formula for discounting

- not only is there a relative price effect
- but the discounting formula itself changes

Discounting in 2 sector model

$$r = \rho + \left[(1-\gamma^*)\alpha + \gamma^* \frac{1}{\sigma} \right] g_C + \left[\gamma^* \left(\alpha - \frac{1}{\sigma} \right) \right] g_E$$

Where γ* is ”utility share” of the environment

$$\gamma^* = \frac{\gamma E^{1-\frac{1}{\sigma}}}{(1-\gamma) C^{1-\frac{1}{\sigma}} + \gamma E^{1-\frac{1}{\sigma}}} = \frac{U_E E}{U_E E + U_C C} = \frac{\frac{U_E}{U_C} E}{\left(\frac{U_E}{U_C} E \right) + C}$$

Comparing discount formulas

$$r = \rho + \left[(1 - \gamma^*)\alpha + \gamma^* \frac{1}{\sigma} \right] g_C + \left[\gamma^* \left(\alpha - \frac{1}{\sigma} \right) \right] g_E$$

$$r(t) = \rho + \alpha g_C(t)$$

Conclusions

- Relative prices CRUCIAL in long run CBA
- Complement discounting by price correction
- Discounting itself is complex in 2 sector model
- Important policy conclusions for Climate
- Next step: integrated GE Climate model

Introducing relative prices into DICE

- Stern has been criticised for low r. δ=0,1 η=1 and per capita g =1,3. Total 1.4
- Nordhaus reproduced Stern-type results with DICE and low r
- We reproduce Stern (or intermediate) results with Nordhaus values (high r)
- By including a small part of non-market sector and changing relative prices.

An even Sterner Review
2 Changes to DICE

Add non market damages & Relative Prices

- The original model maximizes total discounted utility using a CRRA function
- $U(C) = C^{1-\alpha} / (1-\alpha)$
- To include the effect of changing relative prices we use a constant elasticity of substitution function of two goods:
- $U(C)= [(1-\gamma)C^{1-1/\sigma} + \gamma E^{1-1/\sigma}]^{(1-\alpha)\sigma/(\sigma-1)}/(1-\alpha)$

Environmental Damages

- First we assume a share of environmental services in current consumption of 10%.
- We assume damage to environmental amenities will be quadratic in temperature
- At 2,5 °C damage ~ 2% current GDP
- $E(t) = E_0 / [1 + aT(t)^2]$
- *So E is actually falling due to climate ch.*
- *We assume elasticity of Substitution is .5*

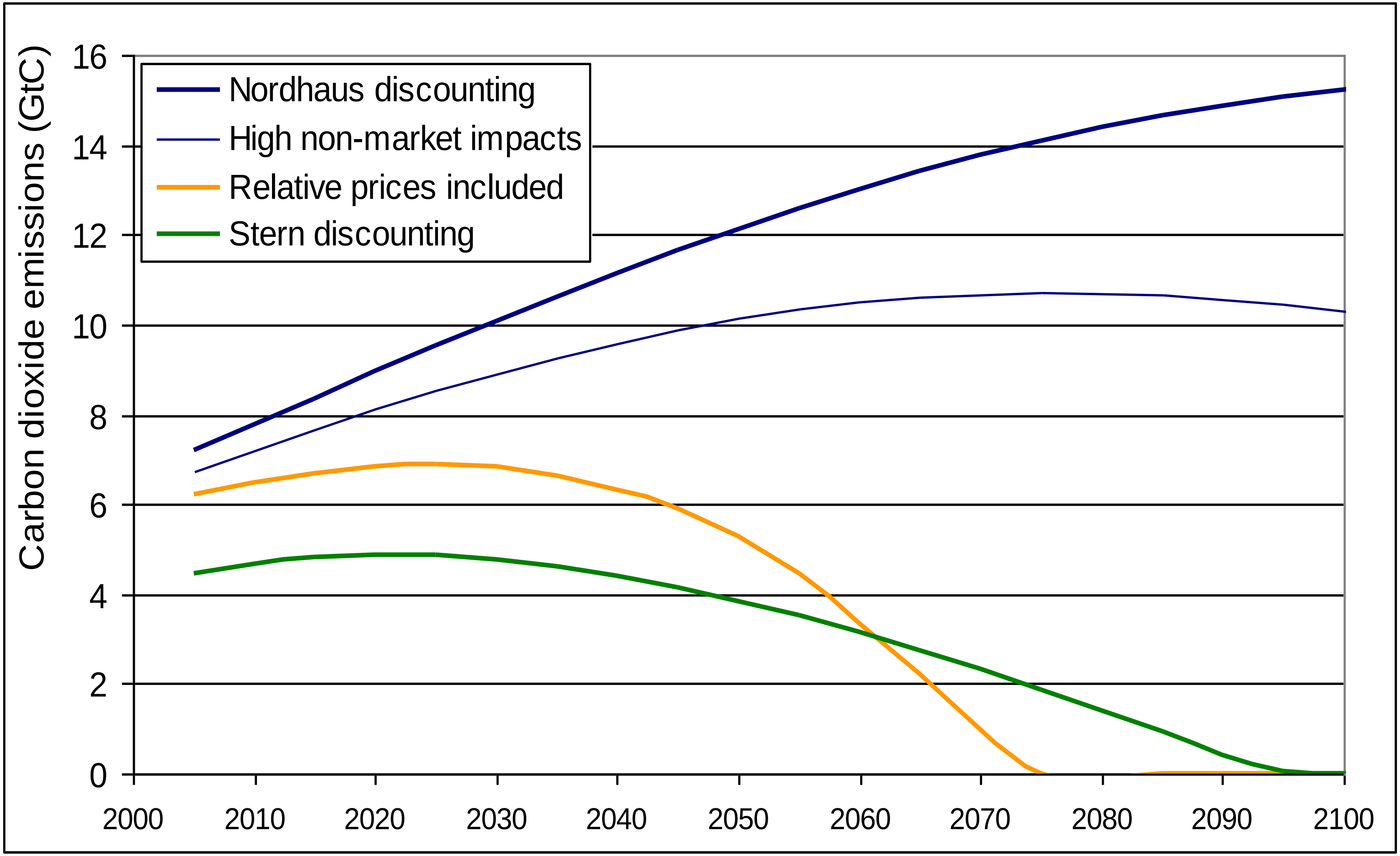


Figure 2: *Optimal carbon dioxide emission paths in the DICE model for four different cases: the original model (Nordhaus discounting), the original model with high non-market impacts(High non-market impacts), the original model with low discount rate (Stern discounting) and a run where the changes in relative prices between market and non-market (environmental) goods is taken into account (Relative prices included). See text for explanation.*

Comparison of discountrates

g_c = 2,5%, rho = 1%, g_E = 0%,

α	σ	Convent r	***2sector R***		
0.5	0.5	2.25	**3.35**		
0.5	1	2.25	**2.37**		
0.5	1.5	2.25	**2.28**		
1	0.5	3.5	**4.24**		
1	1	3.5	**3.50**		
1	1.5	3.5	**3.44**		
1.5	0.5	4.75	**5.12**		
1.5	1	4.75	**4.62**		
1.5	1.5	4.75	**4.60**		

Comparison of discountrates

g_c = 2,5%, rho = 1%, g_E = 0%,

α	*σ*	Convent *r*	***2sector R***	*Price p*	TOT *R*
0.5	0.5	2.25	**3.35**	-5.00	-1.65
0.5	1	2.25	**2.37**	-2.50	-0.12
0.5	1.5	2.25	**2.28**	-1.67	0.61
1	0.5	3.5	**4.24**	-5.00	-0.76
1	1	3.5	**3.50**	-2.50	1.00
1	1.5	3.5	**3.44**	-1.67	1.77
1.5	0.5	4.75	**5.12**	-5.00	0.12
1.5	1	4.75	**4.62**	-2.50	2.13
1.5	1.5	4.75	**4.60**	-1.67	2.94

- Arrow,K., M L. Cropper, C Gollier, B Groom, G M. Heal, R G. Newell, W D. Nordhaus, R S. Pindyck, W A. Pizer, P Portney, T Sterner, R Tol and M,L. Weitzman
  “How Should Benefits and Costs Be Discounted in an Intergenerational Context? “

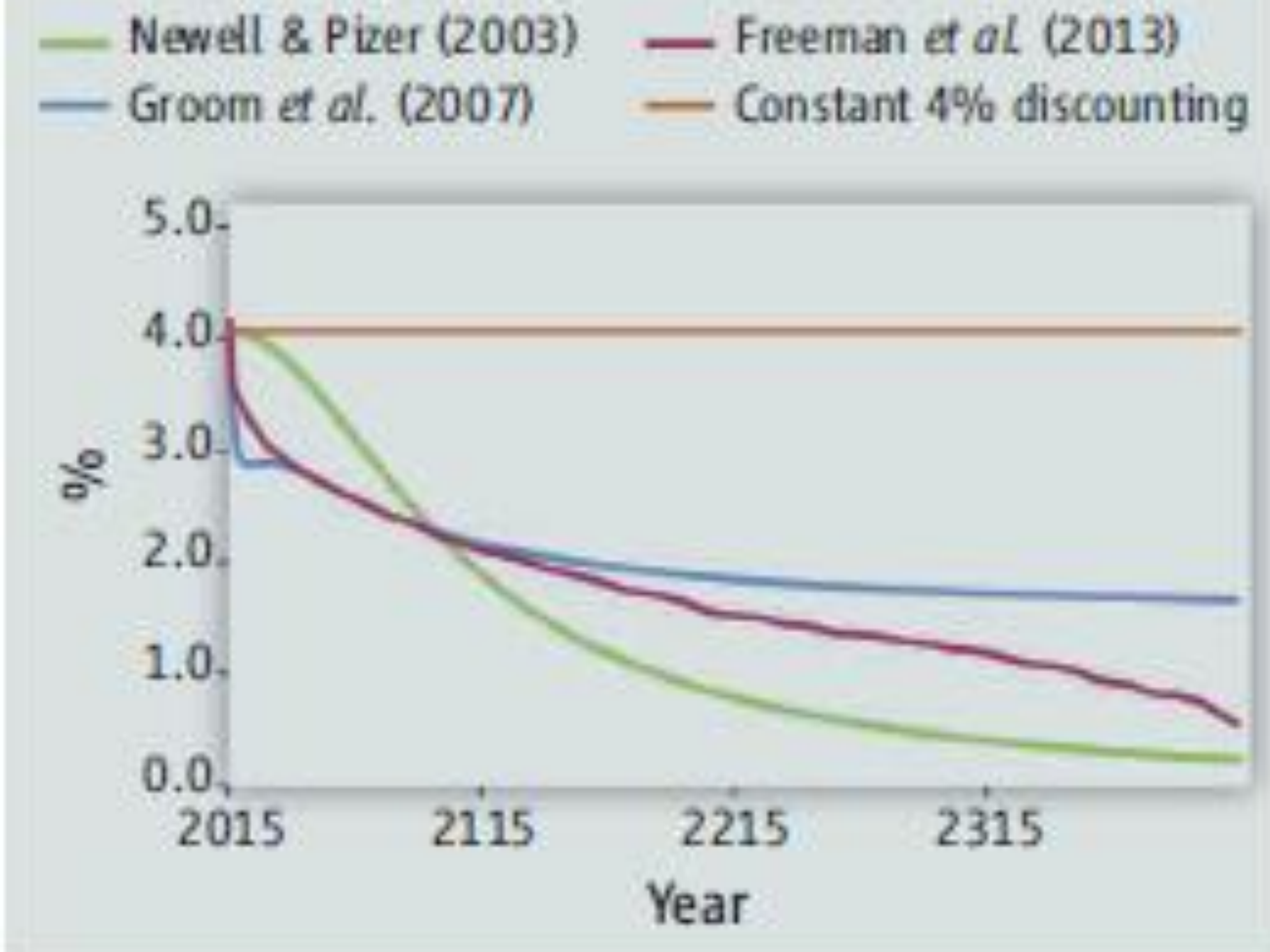
Effect of uncertainty $(1+r)^t$

t	0.01	0.04	0.07	eq P1 or 7	cert eq
1	990.10	961.54	934.58	962.34	3.91
10	905.29	675.56	508.35	706.82	3.53
50	608.04	140.71	33.95	320.99	2.30
100	369.71	19.80	1.15	185.43	1.70
150	224.80	2.79	0.04	112.42	1.47
200	136.69	0.39	0.00	68.34	1.35
300	50.53	0.01	0.00	25.27	1.23
400	18.68	0.00	0.00	9.34	1.18

PRESENT VALUE OF A CASH FLOW OF $1000 RECEIVED AFTER *T* YEARS

t	Value ($) of $1000 at a discount rate of				Certainty equivalent (%)
	1%	4%	7%	Equally likely 1% or 7% expected value	
1	990.05	960.79	932.39	961.22	3.94
10	904.84	670.32	496.59	700.71	3.13
50	606.53	135.34	30.20	318.36	1.28
100	367.88	18.32	0.91	184.40	1.02
150	223.13	2.48	0.03	111.58	1.01
200	135.34	0.34	0.00	67.67	1.01
300	49.79	0.01	0.00	24.89	1.01
400	18.32	0.00	0.00	9.16	1.01

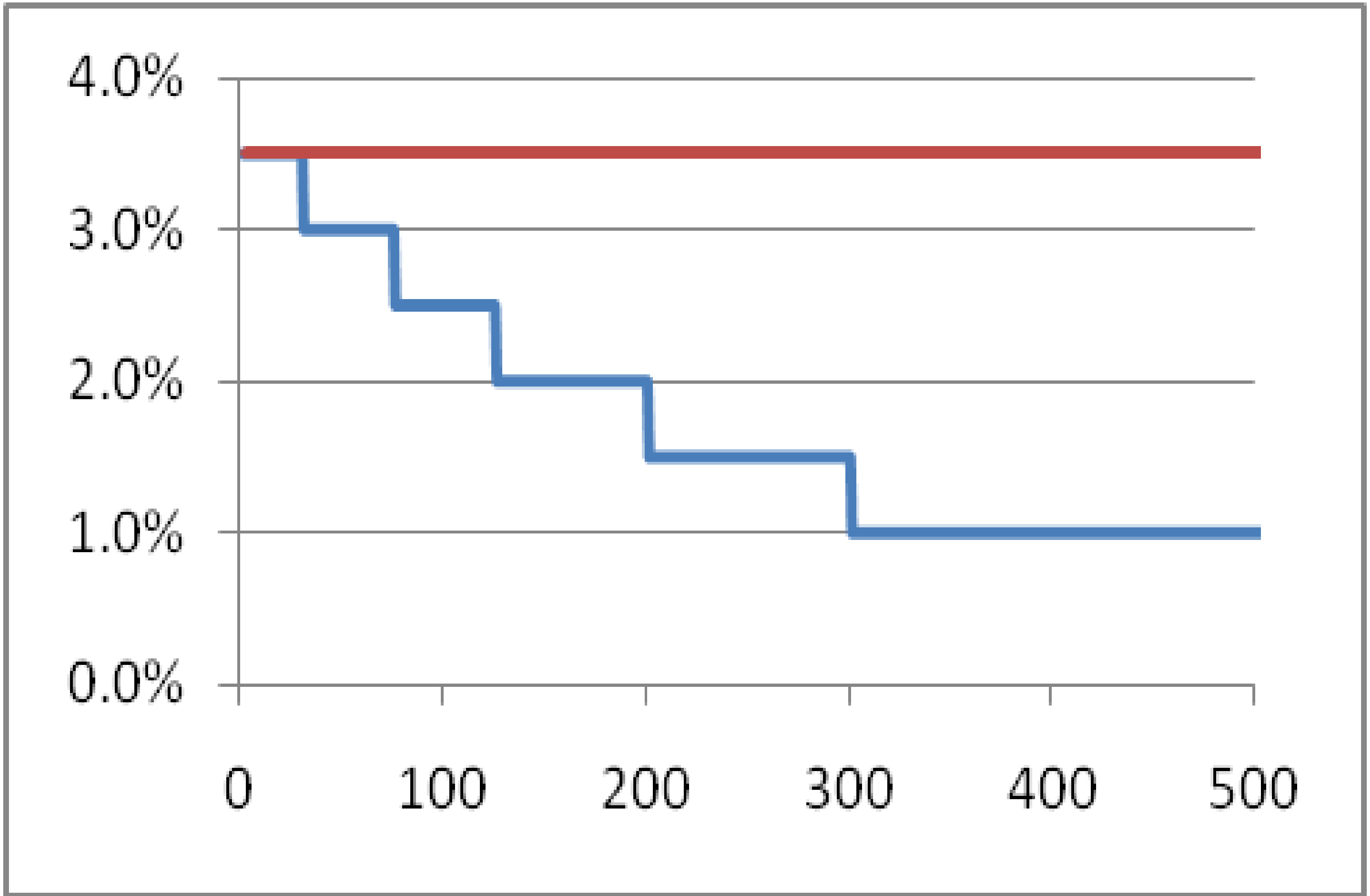
Present value of a cash flow of $1000 received after *t* years. Expected value is the average of values from the 1% and 7% columns.



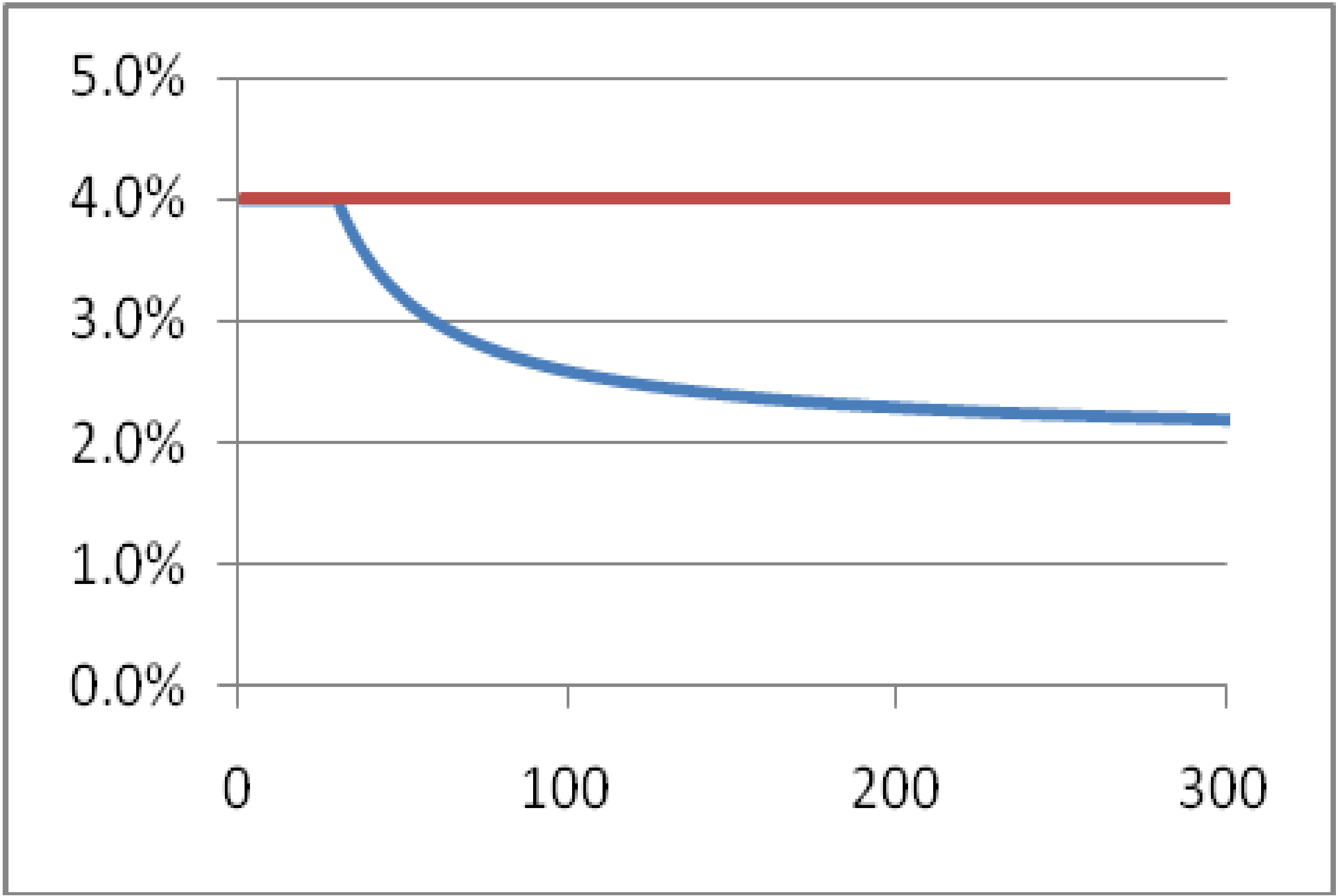
Estimated declining discount rate schedules.
From (*11*, *16*, *17*).

Country	Issuing agency or sector of application	Discount rate	Long-run rate	Theoretical Approach	Reference
United Kingdom	HM Treasury	3.5%	declining after 30 years	SRTP	HM Treasury (2003)
France	Commissariat Général du Plan	4%	declining after 30 years	SRTP	Lebègue et al. (2005)
Italy	Central guidance to regional authorities	5%		SRTP	a
Germany	Bundesministerium der Finanzen	3%		federal refinancing rate	a
Spain	Transport	6%		SRTP	a
	Water	4%		SRTP	a
Netherlands		4%			b
Sweden	SIKA* - transport	4%		SRTP	SIKA(2002)
	Naturvårdsverket - environment	4%		SRTP	Naturvårds-verket (2003)
Norway		3.5%		government borrowing rate	a
United States	Office of Management and Budget	7%	Sensitivity check, >0%	SOC	OMB(2003)
	Environmental Protection Agency	2-3%	Sensitivity check, 0.5-3%	SRTP	EPA(2000)
Canada	Treasury Board	8%		SOC	b
Australia	Office of Best Practice Regulation	7%		SOC	b
New Zealand	Treasury	8%		SOC	b
South Africa		8%		SOC	b
China, People's Republic	NDRC**	8%***	lower than 8%	weighted average of SOC & SRTP	NDRC (2006)
India		12%		SOC	a
Pakistan		12%		SOC	a
Philippines		15%		SOC	a
World Bank		10-12%			Belli et al.

The recommended declining social discount rate in the UK



Recommended rates in France



END or more on rel income..

Now let us turn to behavioral economics

- Suppose we are motivated not just by
- WANTING MORE MONEY

- BUT

- WANTING MORE THAN THE NEIGHBOR

Utility and relative income

$$U_t = u\left(c_t, R_t\right) = u\left(c_t, r(c_t, z_t)\right) = v\left(c_t, z_t\right)$$

Compare du/dc and dv/dc

3 Welfare Functions

$$\underset{\{c_0,\ldots,c_T\}}{\text{Max}} : w^p \equiv \int_0^T u\left(c_\tau, r(c_\tau, z_\tau)\right) e^{-\delta\tau} d\tau = \int_0^T v\left(c_\tau, z_\tau\right) e^{-\delta\tau} d\tau$$

$$\underset{\{c_0,\ldots,c_T\}}{\text{Max}} : w^s \equiv \int_0^T u\left(c_\tau, r(c_\tau, c_\tau)\right) e^{-\delta\tau} d\tau = \int_0^T v\left(c_\tau, c_\tau\right) e^{-\delta\tau} d\tau$$

$$\underset{\{c_0,\ldots,c_T\}}{\text{Max}} : w^R \equiv \int_0^T u\left(c_\tau\right) e^{-\delta\tau} d\tau$$

3 Welfare Functions

$$\underset{\{c_0,\ldots,c_T\}}{\text{Max}} : w^p \equiv \int_0^T u\left(c_\tau, r(c_\tau, z_\tau)\right) e^{-\delta\tau} d\tau = \int_0^T v\left(c_\tau, z_\tau\right) e^{-\delta\tau} d\tau$$

$$\underset{\{c_0,\ldots,c_T\}}{\text{Max}} : w^s \equiv \int_0^T u\left(c_\tau, r(c_\tau, c_\tau)\right) e^{-\delta\tau} d\tau = \int_0^T v\left(c_\tau, c_\tau\right) e^{-\delta\tau} d\tau$$

$$\underset{\{c_0,\ldots,c_T\}}{\text{Max}} : w^R \equiv \int_0^T u\left(c_\tau\right) e^{-\delta\tau} d\tau$$

Comparing Private & Social (in discrete T)

$$\rho^p(t) = -\frac{1}{t}\ln\frac{\partial w^p / \partial c_t}{\partial w^p / \partial c_0} = \delta - \frac{1}{t}\ln\frac{v_{1t}}{v_{10}}$$

$$\rho^s(t) = -\frac{1}{t}\ln\frac{\partial w^s / \partial c_t}{\partial w^s / \partial c_0} = \delta - \frac{1}{t}\ln\frac{v_{1t}+v_{2t}}{v_{10}+v_{20}}$$

SAME if $v_{2t}(c_t) = \beta v_{1t}(c_t)$

PROPOSITION1 Arrow & Dasgupta (2009) prop 1

Intuition Arrow Dasgupta

- Paper that is most akin to ours
- Rat Race: Working & consume more to beat neighbours.
- But this does not necessarily happen because people will be positional in the future too
- Beat Jones's now → Lose in future
- Condition for same optimal path of consumption is

$$v_{2t}(c_t) = \beta v_{1t}(c_t)$$

Defining degree of positionality

$$U_t = u\left(c_t, R_t\right) = u\left(c_t, r(c_t, z_t)\right) = v\left(c_t, z_t\right)$$

$$\gamma_t = \frac{u_{2t}\, r_{1t}}{u_{1t} + u_{2t} r_{1t}}$$

We find same results and more..

- Degree of positionality $\gamma_t = \dfrac{u_{2t}\, r_{1t}}{u_{1t} + u_{2t} r_{1t}}$

$$\rho^s(t) = \delta - \frac{1}{t}\ln\left(\frac{v_{1t}}{v_{10}} - \frac{v_{1t}}{v_{10}+v_{20}}\left(\gamma_t - \gamma_0\right)\right) = \delta - \frac{1}{t}\ln\left(\frac{v_{1t}}{v_{10}}\left\{\frac{1-\gamma_t}{1-\gamma_0}\right\}\right)$$

We find same results and more..

- Degree of positionality $\gamma_t = \dfrac{u_{2t} r_{1t}}{u_{1t} + u_{2t} r_{1t}}$

$$\rho^s(t) = \delta - \frac{1}{t}\ln\left(\frac{v_{1t}}{v_{10}} - \frac{v_{1t}}{v_{10}+v_{20}}(\gamma_t - \gamma_0)\right) = \delta - \frac{1}{t}\ln\left(\frac{v_{1t}}{v_{10}}\left\{\frac{1-\gamma_t}{1-\gamma_0}\right\}\right)$$

- Assume pos growth g>0
- Assume increasing positionality (there is some evidence that dγ/dt>0).
- Then PROPOSITION 2 $\rho^s > \rho^p$

THREE relevant Discount rates

1. The Privately optimal (assuming z unchanged)

2. The Socially optimal (assuming R unchanged)

3. Ramsey Rule which decision makers use

(Private discount rate < Ramsey iff v12>0 – ie iff ”Keeping up with the Joneses)

Comparing 3 discount rates

$$\rho^p = -\frac{1}{t}\ln\frac{\partial w^p / \partial c_t}{\partial w^p / \partial c_0} = \delta - \frac{1}{t}\ln\frac{v_{1t}}{v_{10}}$$

$$\rho^p = -\frac{\partial(\partial w^p / \partial c) / \partial t}{\partial w^p / \partial c} = \delta - \frac{v_{11}}{v_1}cg - \frac{v_{12}}{v_1}cg = \delta + \sigma g - \frac{v_{12}}{v_1}cg$$

$$\rho^s = \delta - \frac{v_{11} + 2v_{12} + v_{22}}{v_1 + v_2}cg = \delta + \sigma g - \frac{v_{12}}{v_1}cg + \frac{d\gamma / dt}{1 - \gamma_t}$$

$$\rho^R = \delta - cv_{11} / v_1 g = \delta + \sigma g$$

Comparing 3 discount rates

$$\rho^p = -\frac{1}{t}\ln\frac{\partial w^p/\partial c_t}{\partial w^p/\partial c_0} = \delta - \frac{1}{t}\ln\frac{v_{1t}}{v_{10}}$$

$$\rho^p = -\frac{\partial(\partial w^p/\partial c)/\partial t}{\partial w^p/\partial c} = \delta - \frac{v_{11}}{v_1}cg - \frac{v_{12}}{v_1}cg = \delta + \sigma g - \frac{v_{12}}{v_1}cg$$

$$\rho^s = \delta - \frac{v_{11} + 2v_{12} + v_{22}}{v_1 + v_2}cg = \delta + \sigma g - \frac{v_{12}}{v_1}cg + \frac{d\gamma/dt}{1-\gamma_t}$$

$$\rho^R = \delta - cv_{11}/v_1 g = \delta + \sigma g$$

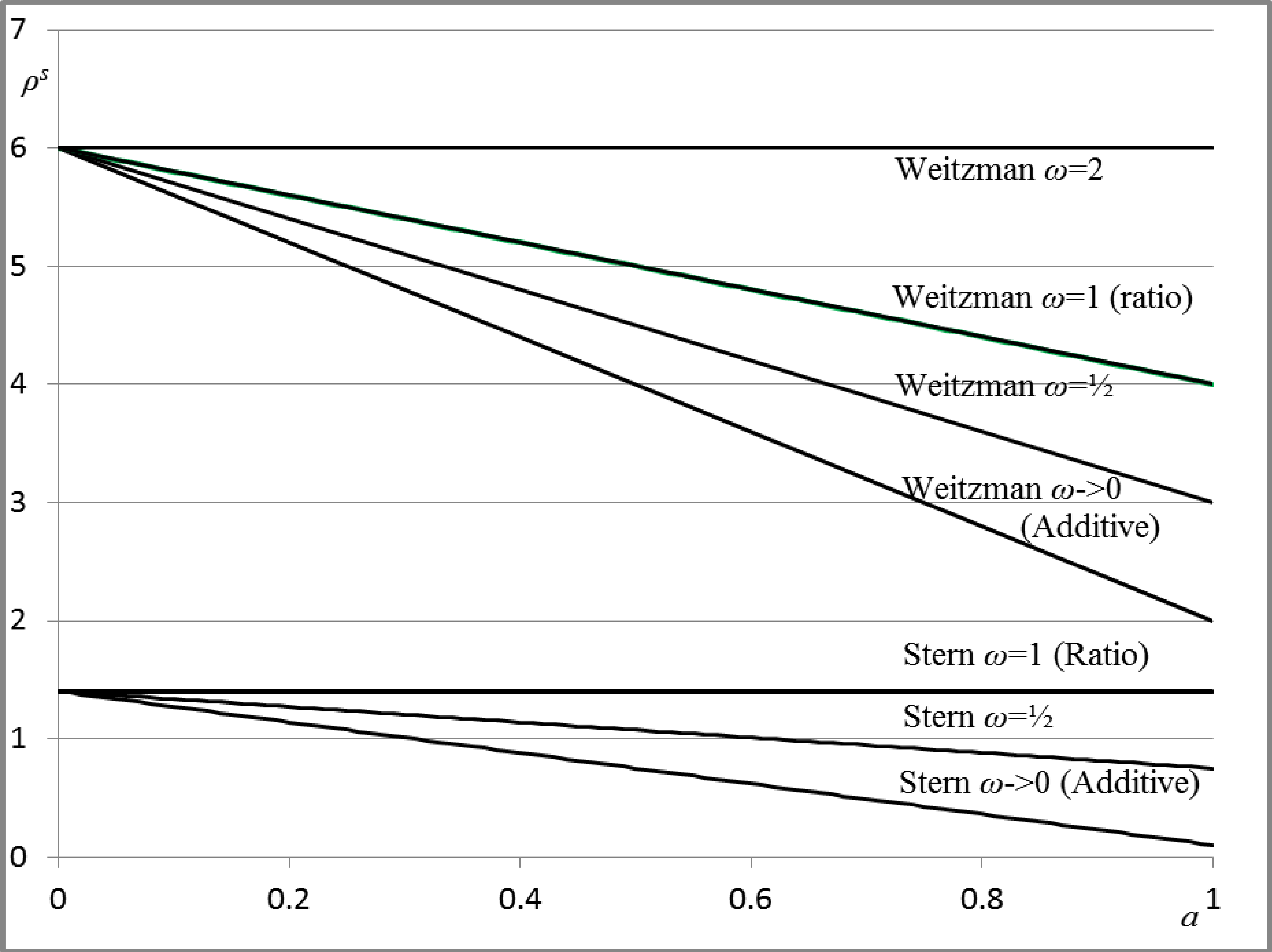
R

J

P

Private < Social < Ramsey

The social discount rate > the private rate but < Ramsey rate if the degree of positionality increases with consumption and preferences reflect risk-aversion with respect to reference consumption and are quasi-concave with respect to own and reference consumption

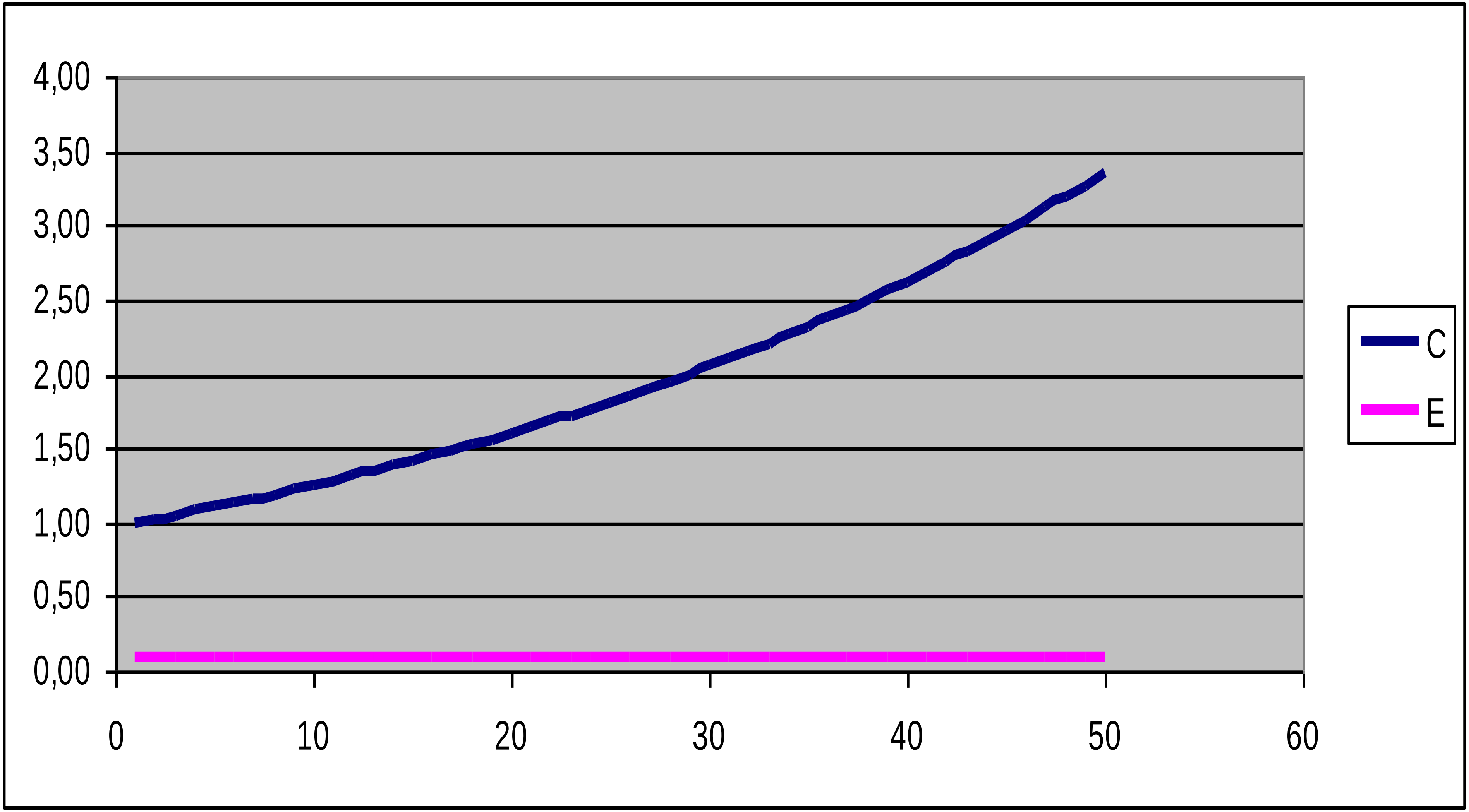


7
ρ^s
6
5
4
3
2
1
0
Weitzman ω=2
Weitzman ω=1 (ratio)
Weitzman ω=½
Weitzman ω->0
(Additive)
Stern ω=1 (Ratio)
Stern ω=½
Stern ω->0 (Additive)
0
0.2
0.4
0.6
0.8
1
a

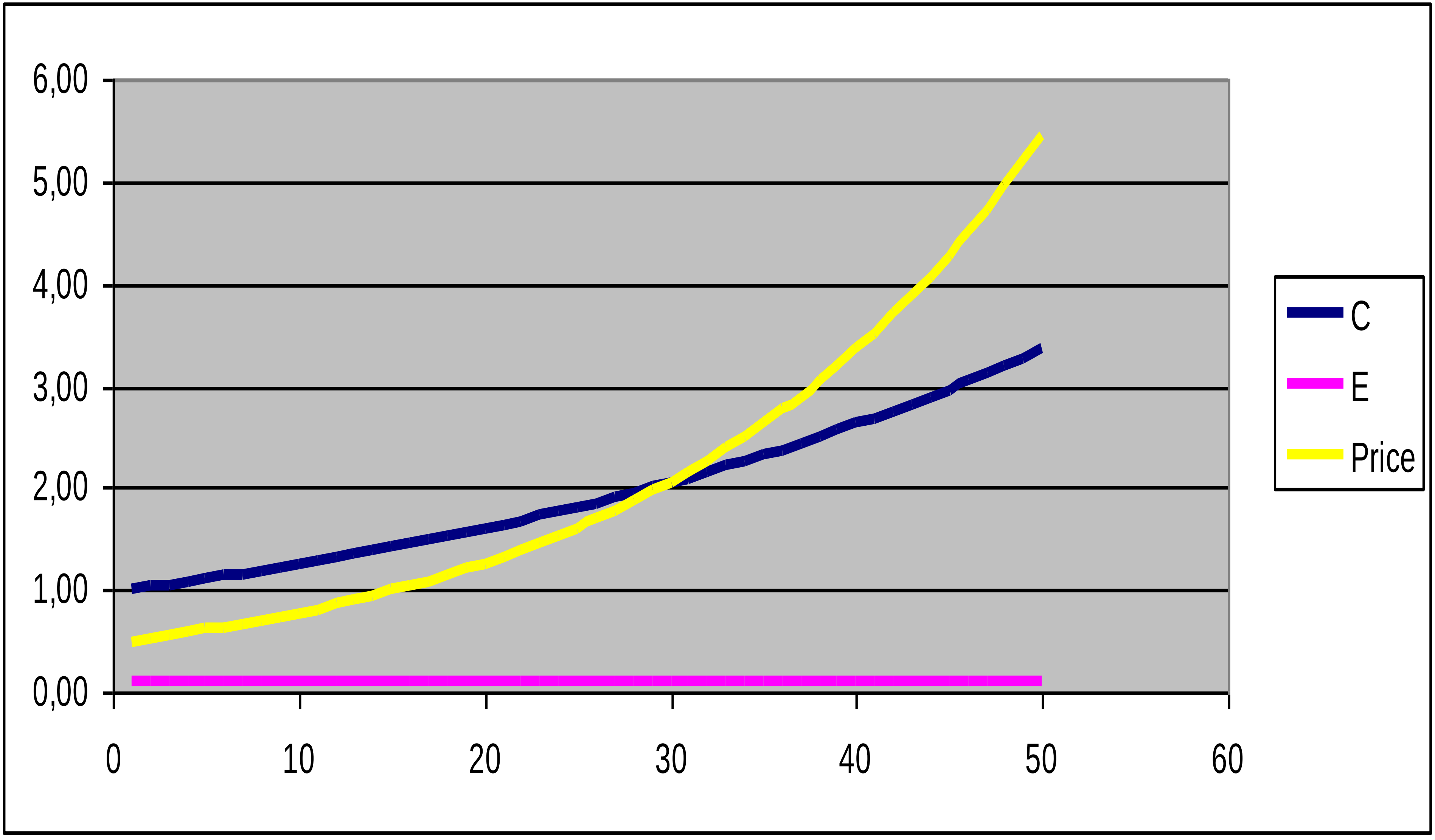
Conclusions

- Social discount rate >= Private
- Equal if $v_2 = \beta v_1$; β is positionality
- Consistent with Arrow Dasgupta (2009)
- Bigger if Positionality increases over time
- This can be internalised through a tax
- Social Rate < Ramsey.
- Implications for Climate change Debate

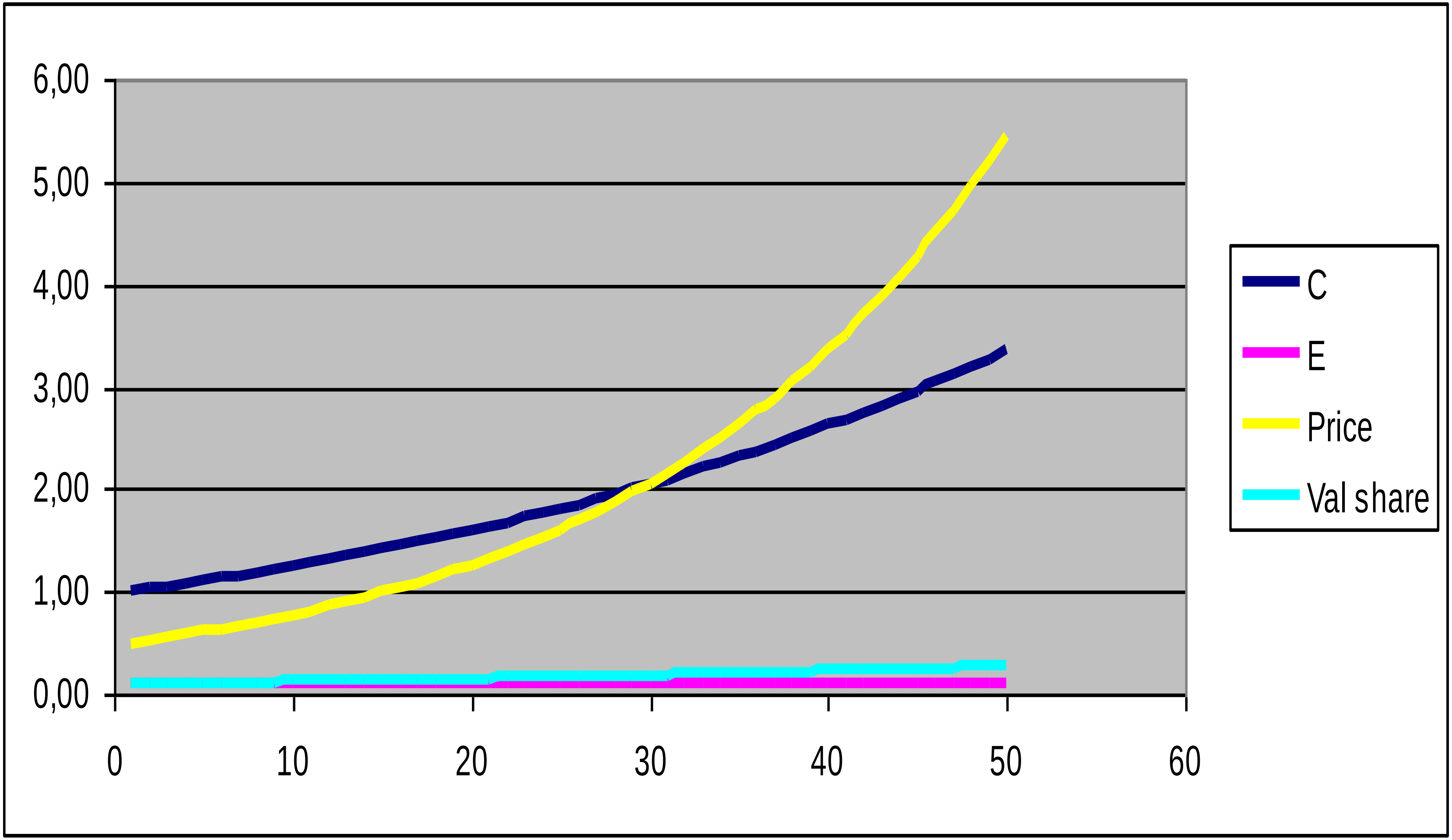
2 sectors, C&E with different rates $\sigma=0{,}5$



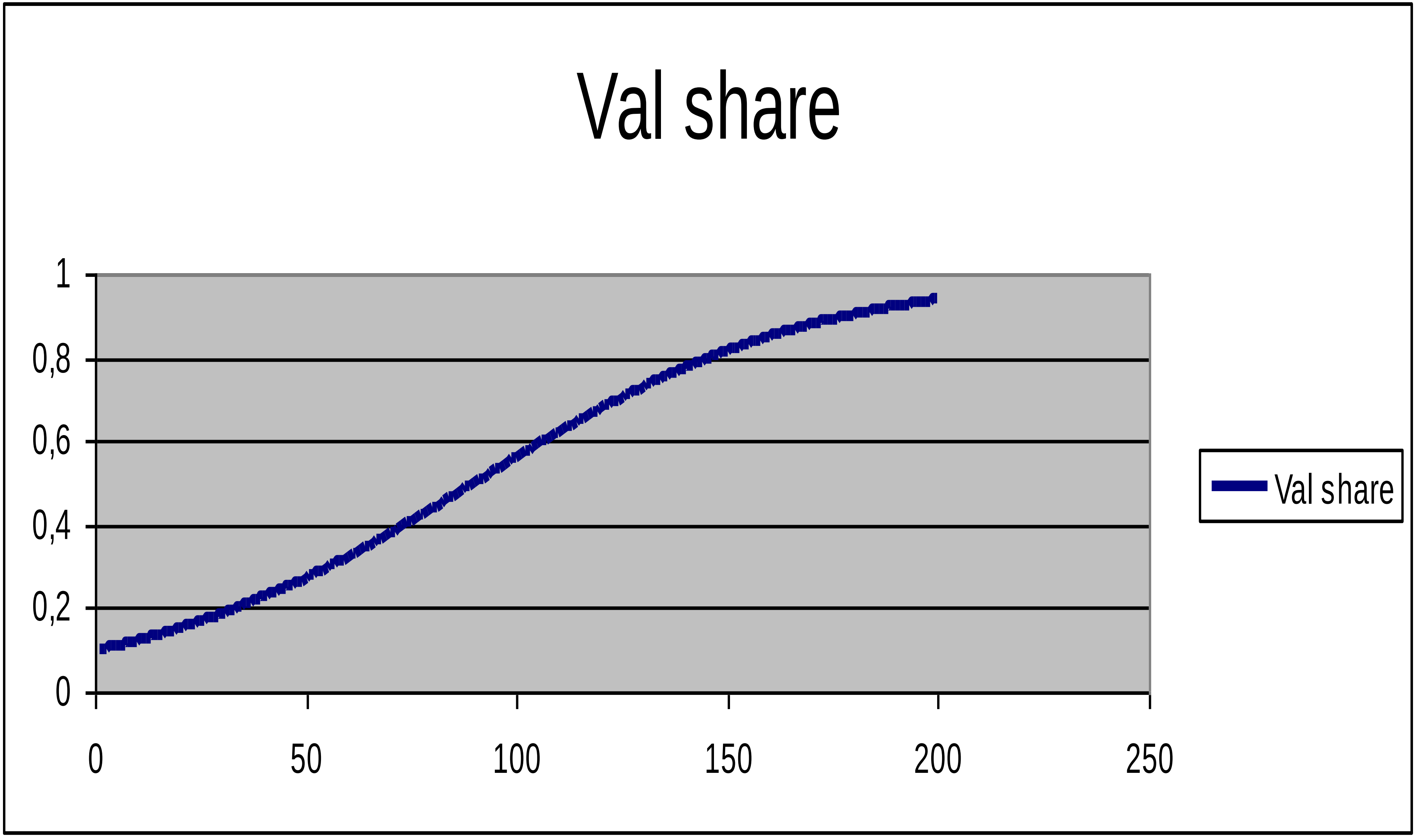
C gets bigger but the price of E goes up FASTER



So the value share of E rises

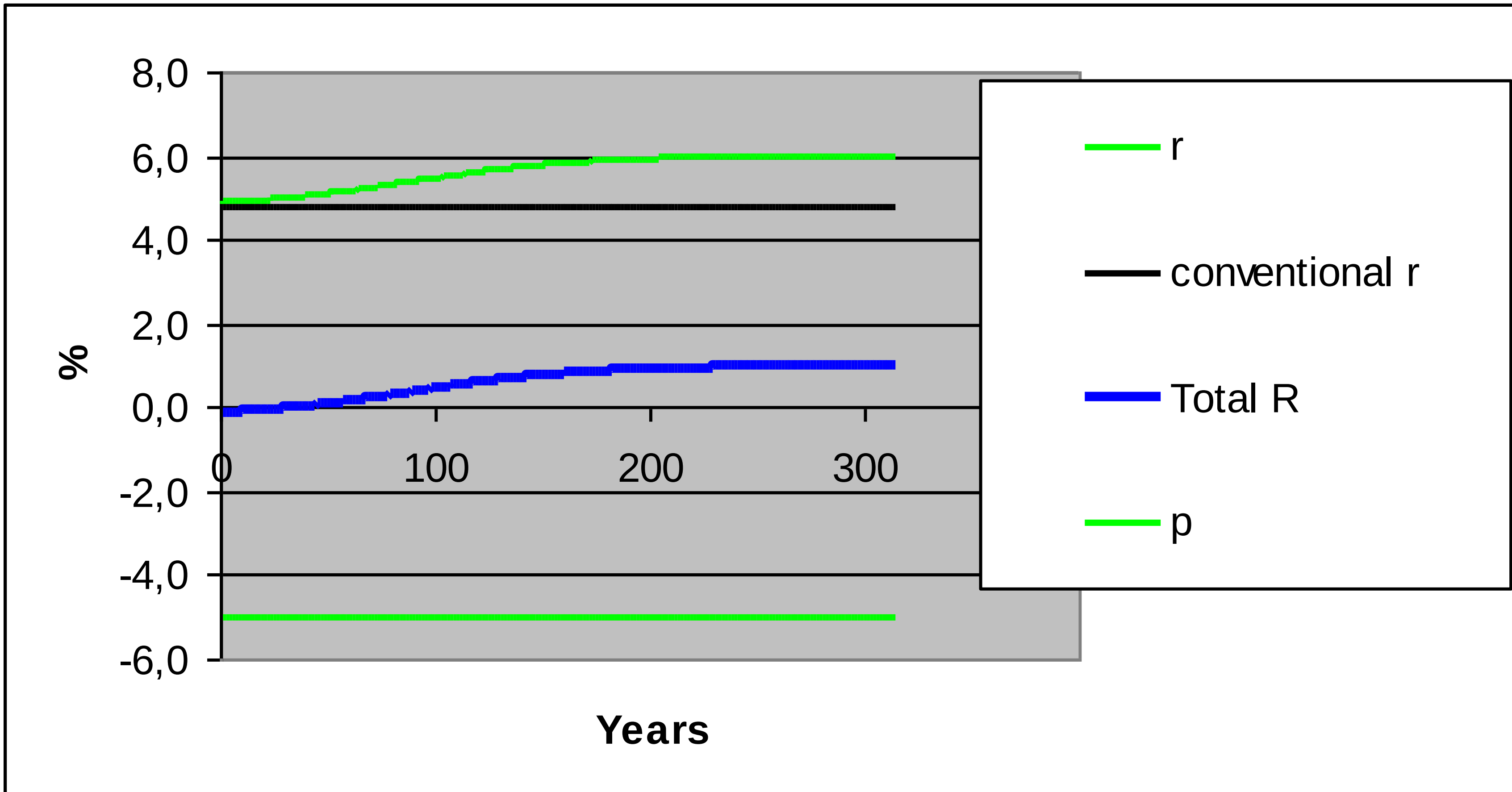


After some time E dominates

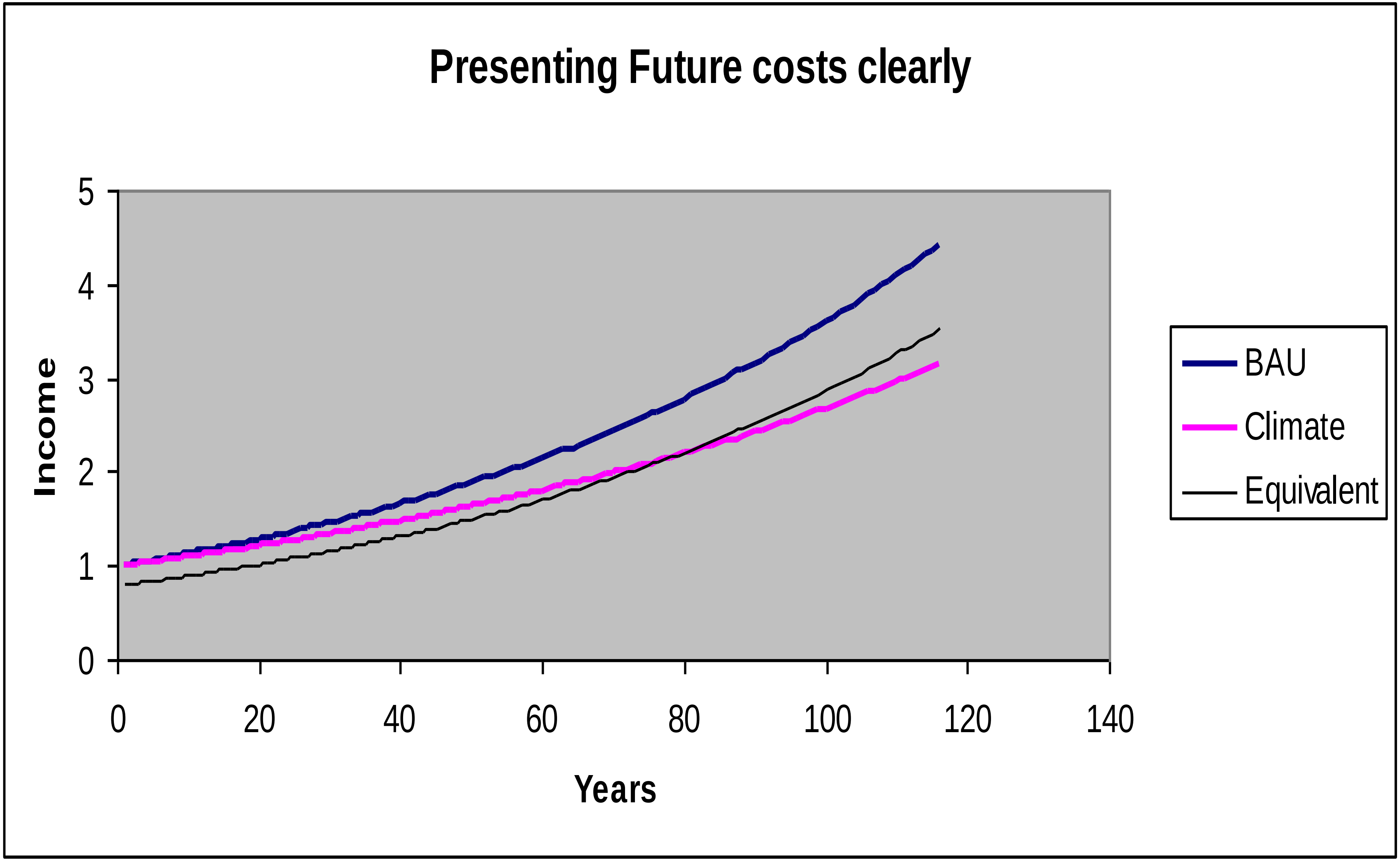


Therefore variation in discount rate

$\rho=0.01, \sigma=0.5, \alpha=1.5, \gamma^*_0=0,1\ g_C=2.5\%$



5-20% For now and forever…



Costa & Kahn, The Rising Price of Nonmarket goods, AEA Papers &P

TABLE 1—THE VALUE OF LIFE IN 2002 DOLLARS, 1900–2000

Year	Value of life
1900	$427,000 (predicted)
1920	895,000 (predicted)
1940	1,377,000
1950	2,426,000
1960	2,884,000
1970	5,176,000
1980	7,393,000
2000	12,053,000 (predicted)